2D and 3D Object Recognition in Subspaces:

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Subspace methods

How to represent
the shape in x?
Point sets,
Depth maps,
Silhouettes,
Geometric measures,
Volumetric methods,
Distance transform,
...

How to design Φ?

Model driven
Data driven

Reconstructive
Dirac, DFT, DCT, ART
PCA, ICA, NMF

Discriminative
LDA, QR

Subspace Methods

• Dirac (Direct Comparisons)
• Discrete Fourier Transform
• Discrete Cosine Transform
• Angular Radial Transform
• Principal Component Analysis
• Independent Component Analysis
• Nonnegative Matrix Factorization

Dirac basis (Direct Comparisons)

• Φ is the identity matrix I.
• The subspace is the original space itself.
• We make direct comparisons, calculate the distance in original Euclidean space.
• Point differences, the differences of attributes of pixels or voxels...
• Dimension may be huge.

Discrete Fourier Transform (DFT)

• Frequency content of signals
• The basis is composed of complex harmonics.
• Filtering, trimming certain frequencies → Project onto a subspace

Discrete Cosine Transform (DCT)

• The basis is composed of cosines.
• Has high energy compaction property.
• If the signal is smooth and highly correlated, DCT summarizes the information in few coefficients.

Angular Radial Transform (ART)

• The basis functions \( \phi_n(\theta) \) are defined in polar coordinates:

\[
\phi_n(\theta) = \frac{1}{2^n} \frac{\sin^n(\theta)}{(\sin(\theta))^n}
\]

\[\mathbb{R}^n \rightarrow \mathbb{R}^r \]

ART-based transform (F):

\[
F = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} e^{-i \left( \theta \phi_n(\theta) \right)}
\]

\[\mathbb{R}^{n \times r} \rightarrow \mathbb{R}^{r \times r} \]
Principal Component Analysis (PCA)

- Decorrelation of data using second order statistics
- Assume the data is Gaussian
- The axes of large variance are assumed to describe the underlying structure.

\[
\begin{align*}
\mathbf{X} & : \text{Data matrix} \\
\mathbf{m} & : \text{Data mean vector} \\
\mathbf{C} = (\mathbf{X} - \mathbf{m})(\mathbf{X} - \mathbf{m})^T & : \text{Covariance matrix} \\
\mathbf{U} & : \text{First K eigenvectors of the covariance matrix} \\
\mathbf{D} & : \text{Corresponding K eigenvalues} \\
\mathbf{A} & : \text{PCA coefficients}
\end{align*}
\]

Independent Component Analysis

- A generalization of PCA
- Removes correlations using high order statistics
- Assumes observed signals are a linear mixture of unobserved source signals: \( \mathbf{X} = \mathbf{AS} \)

\[
\begin{align*}
\mathbf{X} & : \text{Data matrix (actually PCA coefficients after dimension reduction)} \\
\mathbf{A} & : \text{Mixing matrix} \\
\mathbf{S} & : \text{Source signals} \\
\mathbf{W} & : \text{Separating or de-mixing matrix}
\end{align*}
\]

\( \mathbf{Y} = \mathbf{SWX} \)

is estimated via FastICA. Negentropy is used as a measure of independence.

ICA architecture I (ICA1)

- Sources (basis vectors) are statistically independent.
- Coefficients are not.
- Similar to NMF since basis vectors are localized.

ICA architecture II (ICA2)

- Coefficients are statistically independent.
- Sources (basis vectors) are not.
- Similar to PCA since basis vectors are holistic.

Nonnegative Matrix Factorization

- The data matrix is factorized into two nonnegative matrices.
- The basis vectors are sparse and localized.
- Parts based representation.

\[
\begin{align*}
\mathbf{X} & : \text{Data matrix (nonnegative)} \\
\mathbf{W} & : \text{Matrix of nonnegative basis vectors} \\
\mathbf{E} & : \text{Coefficients} \\
\mathbf{Z} & : \text{Data matrix (nonnegative)} \\
\mathbf{W} & : \text{Columns of W from a basis.} \\
\mathbf{E} & : \text{Feature vector}
\end{align*}
\]