1. You are given the following function:

```plaintext
function Q1(X[1:n])
    Input: X[1:n] array of elements with X[i] ∈ {0,1}, 1 ≤ i ≤ n, n is even
    Output: a (an integer)
    b = 1
    a = 0

    for i = 1 to n do
        for j = i to n do
            a = a + X[i]
        endfor
        if X[i] ≠ X[n − i + 1]
            b = 0
        endif
    endfor
    if b = 1 then
        for i = 1 to n do
            for j = 1 to n do
                a = a + a * X[j]
            endfor
            for j = 1 to 2^{n/2} do
                a = a + 1
            endfor
        endfor
    endif
```

Do a fine analysis of the function and express the worst-case \( W(n) \) and average case \( A(n) \) of the algorithm using \( \Theta \) notation. You may consider all the assignment statements in the algorithm as the basic operation and the input as a binary string.
2. Prove or disprove: There exist two non-negative functions $f(n)$ and $g(n)$ such that neither $f(n) \in O(g(n))$ nor $g(n) \in O(f(n))$.

3. Given a function $f(n)$, determine whether the following statements are true or not. Justify your answers formally.

$$f(n) = n^4 \log(n!) + \sum_{i=1}^{5} i^n$$

a) $f(n) \in O(n^5 \log n)$

b) $f(n) \in \Omega(n^5 \log n)$

c) $f(n) \in \Theta(5^n)$

d) $f(n) \in o(5^n)$